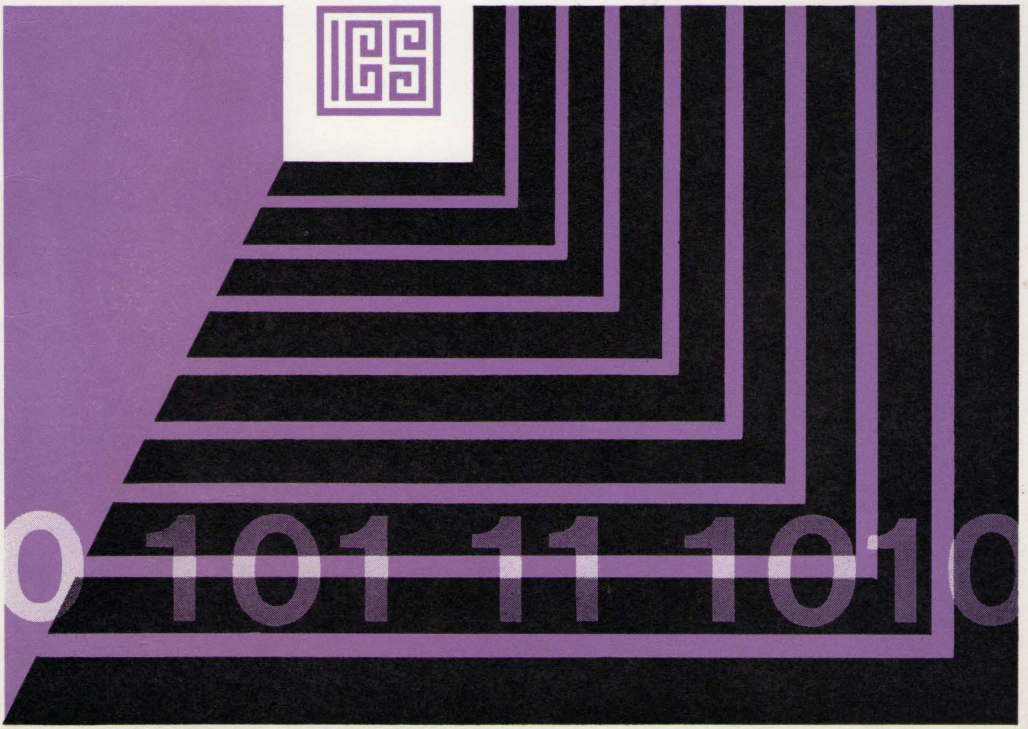
University of London

Institute of Computer Science

Research Report for 1966 and 1967



This is the first research report to be prepared since the Institute of Computer Science was established in 1964 in succession to the University of London Computer Unit. The work reported here refers mainly to the years 1966 and 1967 but in the nature of things much of it has roots in earlier years, and has been described in Annual Reports of the Institute. A further report will shortly be issued for 1968.

Research at the Institute is broadly in three main fields, the first being concerned with the basic problems of computing systems. These may be problems of design, of control, of communication between man and machine, or of theoretical properties of systems. The facilities available have precluded any attention to hardware design, and the main effort has therefore been concerned with programming languages and the development of software.

The second field is that of applications, which in our case largely arise from problems of operational research and statistics, although as this report shows many other applications have been studied at various times. Thirdly, numerical analysis and the development of numerical techniques are the concern of a group which had its origin in the Computer Unit.

The growth of research in these areas naturally owes a great deal to the presence of an Atlas computer at the Institute since 1964. Its continued development in the present state of University financing depends to a large extent on the availability of research grants, both for equipment and staff. Thus the installation of a PDP 9 system with the help of the Science Research Council is already providing a stimulus for several new projects besides that for which it was acquired. The importance of keeping the accessible computing facilities up to date cannot be stressed too strongly.

The human side is of course of paramount importance. Many projects, particularly those concerned with the computer systems, are carried out by small groups led by established Institute staff; many others essentially follow the particular interests of individuals, and the report tries to make this clear. As in most university departments* engaged in research the people involved include established academic staff and research assistants, programming assistants, research staff employed on grants, visiting fellows, full and part-time research students, and also temporary staff who may be vacation or sandwich students. The total number involved in 1967–68 was about 70. The fact that research at the Institute has reached this level of activity in a relatively short time is a good augury for future progress.

R. A. Buckingham
Director

*The status of many members of the Institute is indicated by the following symbols: ‡ grant-supported staff; * short-term assistant; † research student.

Man/Computer Systems

DESIGN OF PROGRAMMING SYSTEMS

B. Higman
J. N. Buxton
D. F. Hendry
R. J. Housden
E. Nixon
W. L. B. Nixon
Abigail J. Davidson
† G. M. Bull
* N. Dyson
W. S. Hilder
Diana Lawrence
† W. Tagg

† research student
* short-term assistant

CPL
E. Nixon
J. N. Buxton
G. F. Coulouris
* T. J. Goodey
Roberta W. Hill
R. W. Keeling
* D. Levin
R. J. Wakefield

BCL
D. F. Hendry
D. R. Brough
* R. J. Cocking
* Ljubica Hrnjakovic
Sonia M. Lyons
Anne L. Meredith
B. Mohan
Leila Prince

CRAMPON
E. Nixon
W. D. Bell
R. J. Wakefield
Denise Cutler

The main concern of this group, one of the largest in the Institute and in practice a composite one containing several subgroups, is the design and development of programming languages and their implementation as compiler systems. This has been a major activity since the Institute was formed in 1964 and follows earlier work on the CHLF version of Mercury Autocode, which has itself been superseded by the current programming system of EXCHLF.
With two exceptions the following projects have been carried out entirely without external support.

The development of CPL

The CPL project began in 1962 as a collaborative venture by the Computer Unit and the Mathematical Laboratory, Cambridge. The initial motivation was to provide a general-purpose language for the Atlas machines at London and Cambridge, but as work on the project proceeded the emphasis shifted from the initial pragmatic approach to a more theoretical and rigorous analysis of the principles underlying programming languages. The basic concepts of CPL were outlined in the joint paper by D. W. Barron and others in 1963.

By the first half of 1966 the work progressed to the point where an initial version of the definitive language description was written and circulated under the title 'CPL Working Papers' as a technical report from the two universities concerned. This had considerable success in stimulating wider interest in the language. Further work on some aspects of the language remained to be done, particularly in the fields of data structures and program segmentation, and recent developments have been based mainly on C. Strachey's research group at Oxford.

Implementations of the language were carried out independently in London and Cambridge. At the Institute a compiler was developed using the techniques of the Compiler Compiler by G. F. Coulouris and other members of the group, and led to a working compiler for an early version of CPL towards the end of 1965. This was subsequently improved, and list-processing facilities were added. This compiler proved valuable as an experiment in the use of CPL although it has not so far been developed into a full-scale production compiler.

PUBLICATIONS

D. W. Barron/J. N. Buxton/D. F. Hartley/E. Nixon and C. Strachey.
The main features of CPL. Comput. J. (1963) **6,** *pp* 134–42.
G. F. Coulouris/T. J. Goodey/Roberta W. Hill/R. W. Keeling and D. Levin.
The London CPL 1 *compiler. Comput. J.* (1968) **11,** *pp* 26–30.

OTHER REPORTS

R. W. Keeling.
List processing routines in CPL. Tech. Rep. 6 *January* 1966.
G. F. Coulouris and T. J. Goodey.
The CPL 1 *system manual* (*2nd edition*). *April* 1966.
J. N. Buxton/G. F. Coulouris/D. F. Hartley/E. Nixon and others.
Technical Report—CPL Working Papers (*Elementary programming manual, Reference manual*). *July* 1966.

The BCL project

The BCL group is engaged in the design of the BCL language and the writing of a generalised compiler system to implement BCL on a wide range of computers. BCL is a general purpose language

covering mainly the three fields of commercial programming, numerical computation and the writing of compilers using syntax-directed methods.

The group gratefully acknowledge the encouragement and financial support for part of the work received from the Atlas Computing Service and Medical Research Council.

(a) Implementation of BCL compilers

The objective here is to make the language available in a standard form on as wide a range of machine as possible. The BCL compiler is in fact written in BCL, and implementation on different machines is based upon the principle of isolating the machine-independent parts of the compiler and rewriting only the machine-dependent parts.

A subset of BCL, sufficient to be usable as a syntax-directed compiler, has already been implemented on Atlas I. Work is almost completed on a BCL compiler for IBM 360 and SDS 9300 computers, the latter being undertaken by MRC staff. A start has also been made on compilers for ICT 1900 and Elliott 4100 machines.

(b) A list-processing language LSIX has already been implemented in terms of BCL, and is operational on Atlas.

(c) Further developments will include the writing of Algol and Fortran IV compilers in BCL; also the writing of special-purpose diagnostic mode compilers for use on an interactive conversational basis.

INTRODUCTORY REPORT

D. F. Hendry.
A manual of the Atlas Commercial Language (ACL). September 1965.

PUBLICATIONS

D. F. Hendry.
The handling of data structures. PL/1 symposium: National Physical Laboratory. May 1967.

D. F. Hendry.
The BCL programming language. Datamation, December 1967, *pp* 39–40.

B. Higman.
The development of BCL. Computer Weekly, 29 *February and* 7 *March* 1968.

R. J. Housden.
The definition and implementation of LSIX in BCL. (*Submitted for publication, March* 1968.)

BCL WORKING PAPERS

D. F. Hendry.
Provisional BCL manual. ICSI 106. *November* 1966.

B. Mohan.
BCL 1 *manual. ICSI* 103. *February* 1968.

D. R. Brough.
Provisional specification of the intermediate code. July 1967.
Specification of the program resident in the target machine. February 1968.
Specification of the interface. March 1968.

Crampon

Arising out of the study of the computing process (referred to under Theoretical Systems page 9) a language is being developed by means of which one can express the semantics of a programming language. It is hoped that by means of this language the designers of high-level languages and systems will be able to express the total meaning of the various constituents of their languages or systems in a machine-independent and unambiguous fashion.

Moreover it is hoped that the system will be sufficiently powerful for this description itself to be the compiler for the given language (or implementation of the system). Also it would seem that descriptions of computing machines could be expressed in the same notation so that it should be possible to express compilers for any given machine completely within the system.

Work is at present in progress upon the first implementation of a prototype of this language. This is being written for a UNIVAC 418 and when it is completed it is hoped to use the 418 system to transfer the compiler to ATLAS.

PUBLICATION
E. Nixon.
Crampon: A preview of the software of the future? I.D.H. Seminar. March 1968.

REPORT
R. J. Wakefield.
The scope of Crampon. February 1968.

The EXCHLF system

The development of the EXCHLF system to its present stage has been an exercise in making more general and rational the original facilities of CHLF, which was itself a product of Mercury Autocode. This has been done without introducing incompatibilities or appreciably reducing the efficiency and compactness of the earlier system. Although this has limited the scope of what could be done, it has been demonstrated that despite such limitations much improvement is possible.

The most remarkable feature of the present EXCHLF is perhaps the generalised and unified provision for input and output known as the 'chi-system'. This is an attempt to assimilate the varying properties and conventions of different input/output mechanisms and the great variety of card and tape codes in actual use. A separate set of facilities allows for true binary input and output, as distinct from the use of non-standard character codes.

One characteristic of the EXCHLF development has always been the sensitivity to users' needs; thus many substantial additions have been made in response to users' suggestions or to counter their difficulties. The project has shown the real value to the

University of a programming system under the control of university staff. Copies of the complete printout of the compiler, written in Atlas Basic Language (ABL), are available for study.

A by-product of the system is a program known as DICDOC, written in EXCHLF, which permits information relating to the distribution and costing of documents (or other items) to be stored on magnetic tape, and made available on request. One form of output from the program consists of printed address labels for selected recipients.

MANUALS

W. L. B. Nixon.
CHLF Autocode Handbook. March 1965.
W. L. B. Nixon.
Supplement to CHLF Autocode Handbook: EXCHLF extensions. September 1965.
W. L. B. Nixon.
EXCHLF Autocode Handbook. 1967–68.

TECHNICAL NOTES

A. Fairbourn.
Atlas CHLF compiler: Technical information. January 1965.
A. J. Davidson.
Atlas EXCHLF (1967)*: Technical information. LSP* 56. *September* 1967.
A. J. Davidson.
EXCHLF Autocode: Advanced input/output facilities. LSP 57. *October* 1967.

Development of linguistic principles

A somewhat loosely constituted activity is centred around the nature of processes which involve the use of language, including the structure of the units which interpret language. The subjects range from logic design and machine codes through programming languages and their implementation to certain aspects of natural languages. Formal languages form a theoretical background to this, but are not actually 'used'; implementations of programming languages are relevant only insofar as they are 'natural', and natural languages just so far as they provide, in syntactic and semantic analysis and generation, features which are imitable with reasonably limited equipment.

This sphere of activity, under the general direction of B. Higman, is at an early stage of development. Preliminary mention should be made of an investigation by W. S. Hilder of an unusual aspect of information retrieval, namely, how a system which stores certain statements of fact can in some way be 'aware' of deductive implications of those facts. An M.Sc. student, Nguye Phuc Toan, has made a study of logic design requirements for convenient input handling of information consisting of bit strings of variable length. A system called Textile is under development which is based on the (anthropomorphic) assumption that input inform-

ation is never forgotten but must be handled by reference to arbitrary segments obtained by analysis of the input material.

INTERNAL REPORTS

B. Higman.
The Compiler Compiler principle as a guide to future developments in syntactic analysis.
W. S. Hilder.
A project: The implementation of a deductive question-answering system. October 1966.

Simulation languages

The language CSL (Control and Simulation Language) was first developed by Buxton and Laski in 1962. Some extensions and applications of the language have been made subsequently, and reported in the following publications:

J. N. Buxton.
Writing simulations in CSL. *Comput. J.* (1966) **9,** *pp* 137–43.
Basic principles of simulation languages. Calculo (1966) **3,** *supplement* 1, *p* 11.
A control and simulation language. Calculo (1966) **3,** *supplement* 1, *p* 35.
J. N. Buxton and A. T. Clementson.
Compiling strategies for some CSL implementations. Proc. 4th Int. Conf. on Oper. Res., Camb., Mass. August–September 1966.

On-line system for education

This project has aimed to produce a hardware-software combination suitable for making experiments in the educational use of an on-line system, with particular reference to a joint development between Hatfield College of Technology and the neighbouring Hatfield School. The areas of experimentation are computer-aided instruction and the design of a programming system suitable for teaching purposes. The system is based on an Elliott 803 B computer, supplemented by teletypes and a slide projector with carousel slide tray.

The on-line programming system (HSL) uses a language similar to Dartmouth Basic, and incorporates a command interpreter and filing system. With the diagnostics included this system is closely akin to a programmed learning system.

The C.A.I. packages include one program to simulate a console session in the on-line system with the help of a teletype, and another which uses both teletype and slide projector and demonstrates the ability to display graphical information, and to control its display according to the response of the students. Other programs being implemented will enable C.A.I. programs to be written more easily, and also monitor the work of students.

This project is being carried out by G. M. Bull (Hatfield College of Technology) and W. Tagg (Hatfield School).

OPERATING SYSTEMS

A. Fairbourn
‡ C. Bardell
‡ P. F. T. Grant
‡ N. J. Martin
E. Rejwan
J. R. Southey
‡ Patricia A. Ceely
‡ Carolyn Canney
‡ C. White

‡ grant-supported staff

This group is interested in the design and implementation of multi-programming operating systems. Their effort is now directed particularly to the machine-independent aspects of operating system design, and to the use of machine-independent machine code as a means of implementing suitable parts of operating systems and associated compilers.

The work of the group has been extensively supported since August 1965 by the Science Research Council, under grant B/SR/2439.

Examination of the ICT Atlas Supervisor at work

A section has been added to the ICT supervisor program to provide information about its internal operation during normal production use. The statistics collected show the distribution of use of the central processor between various activities of the supervisor and user programs, and the use of peripheral equipment. A detailed analysis has also been made of the cause of idle time during the execution of a normal job mix.

A new experimental Atlas Supervisor

A new supervisor program is being written for Atlas with the aim of experimenting with new ideas in the internal organisation of the program. The external features of this supervisor, as seen by the user, are almost identical with the standard ICT version, thus allowing test jobs to be taken from the normal machine workload. The main areas of research concern the scheduling of machine activities and resources, particularly space and time scheduling. One or more magnetic tapes are used as random access block files for the storage of input files for user programs, output from user programs, and system information; a flexible general purpose transfer system has also been developed to move information about between core store, drums and file tapes.

The new supervisor is expected to be fully developed and operational before the end of 1968. This result owes a great deal to the unstinted cooperation of the ICT software and engineering teams.

An on-line typewriter is in course of commissioning, and will be used for operator-machine communication.

Machine-independent programming

A Machine Independent Machine Code Language (MIMCOL) is being developed to facilitate the transfer of programs between

different computers at machine code level. Thus the system may be used to transfer parts of an operating system, machine-independent compilers or user programs. When applied to complete programs a hypothetical computer LUNAC is defined, with an order code which is a particular form of MIMCOL. A simulator for LUNAC on Atlas is in course of testing, and an Algol compiler is also being designed. Simulators for other computers are under consideration.

A further logical development of the work on MIMCOL, and of experience with the Atlas and other operating systems, is in the direction of machine-independent operating systems which will allow programs in high level languages to be transferred without change from one computer to another. Preliminary work in this field will be undertaken during 1968.

INTERNAL REPORTS

A. Fairbourn.
Statistics on the performance of Atlas and its supervisor. February 1965.
A. Fairbourn.
A survey of the space and time execution efficiencies of Atlas compilers. November 1966.
A. Fairbourn.
Working paper: Specification of the LUNAC computer. April 1964.
A. Fairbourn.
Working paper: Provisional LUNAC III computer manual. February 1968.

THEORETICAL SYSTEMS

The interests of this group lie in two closely related fields:
(1) computability, including aspects of the theory of formal languages;
(2) the nature of the computing process.
The group is of quite recent formation, and much of its effort has been given to the development of a postgraduate course in the theory of computation. However, it is already possible to forecast promising lines of research.

M. G. Bell
R. S. Bird
E. Nixon
R. J. Wakefield

The computational power of automata

R. S. Bird is investigating the computational power of certain families of automata. He has extended a technique to show that certain sets of numbers written in binary notation cannot be recognised by machines with a single pushdown store.

The computing process

The concept of an 'oracle' is being applied by E. Nixon to examine the nature of the computing process. Using results so far obtained, he and R. J. Wakefield have designed a system within which the semantics of computer programs may be expressed. An implementation of this system has begun, and this may serve as a semantic-oriented Compiler Compiler.

COMPUTER COMMUNICATIONS AND GRAPHICS SYSTEMS

P. T. Kirstein
‡ A. R. Duncan
‡ I. H. Gould
‡ D. Lalla
‡* G. England
† T. Fenner
‡* H. Gomaa
‡* J. Wielgosz

* short-term assistant
† research student
‡ grant-supported staff

The main responsibility of this group is to develop techniques for utilising large remote computers linked to local devices which may include satellite computers. The field of investigation includes many problems of intercommunication, and the design of a flexible and varied local system suited to the needs of teaching and research in computer science.

The activities of this group have been made possible by the support of the Science Research Council under grant B/SR/3050, which includes the provision of a small computer. This support is gratefully acknowledged.

The INDRA project

In the Institute Display and Remote Access Project (INDRA) a PDP 9 with a CRT display will be linked over telephone lines to a large 360/75 computer at the Rutherford Laboratory. The project will develop techniques for having occasional interaction with a large scientific design program, run on the IBM 360, to allow the variation of parameters in an on-line mode as a result of seeing pictures of the computer output on the local display.

The project centres around the remote running of the one particular application, but general-purpose programs will be developed to facilitate later extensions. In particular the group is investigating the type of facilities (hardware and software) required for the man-machine interface and the limitations imposed by telephone-line telecommunication between the large computer and the small local computer for graphical output. A set of useful routines will be developed to allow other similar sorts of programs to be run on the large machine, with the input/output manipulated on the small machine. These facilities will be designed to be used easily by a physicist or engineer who is interested in solving specific problems rather than by a computer specialist.

The project started in July 1967 although the equipment was ordered earlier; the group reached full strength only at the end of October 1967. Because the computer was late in arrival and the software not yet available, the main effort in 1967 was given to developing the applications program on the IBM 360/65 at University College London. This was running satisfactorily by the end of the year. In addition careful thought has been given to what modifications would be required in the software supplied by the manufacturer.

The hardware ordered included a basic PDP 9 with 8 K store, extended arithmetic unit, 3 DEC tape units, graphical CRT display with light pen, memory protection and communications interface to the telephone line. Delivery is scheduled for early 1968.*
Initially communication with the 360 machine will be at the slow data rate of only 600 bits per second, but towards the end of 1968 the telecommunications will be replaced by a faster system at 2400 baud. The first on-line experimental runs are expected to take place during the summer of 1968. It became clear at an early stage that it would be necessary to have a random access backing store which is faster than tape, and in September 1967 application was made to the SRC for a 131 K word drum.

Detailed descriptions of the work of the group are given in a number of Internal Memoranda which are listed below.

LIST OF INDRA INTERNAL NOTES–1967

No. 1 General description of the project.
No. 2 PDP 9 configuration and delivery.
No. 3 Simulation of the IBM 1070 by the small computer.
No. 4 Changes in electron beam program required for running on the 360.
No. 5 Proposed communications link at Imperial College.
No. 6 Use of PDP computers for the rest of 1967.
No. 7 Notes on PDP 9 basic software.
No. 8 Basic software routines in the PDP 9 available at Queen Mary College, Department of Nuclear Engineering.
No. 9 Use of logical numbers in the electron beam program.
No. 10 Subroutines ADDPAR, STRPAR, LONPAR, SCRAT, PAK and UNPAK.
No. 11 Data entry and function keyboards.
No. 12 The job control language of the Operating System 360.
No. 13 Punching EBCDIC on the 026.
No. 14 Philosophy of the PDP–IBM link.
No. 15 Format for messages down the link.
No. 16 PDP 9 to Atlas Communication ADAPTER (PACA).
No. 17 Acceptance tests for ULICS Indra equipment.
No. 18 Input format and user data service for the beam program.
No. 19 Application of the electron beam program to ion guns.
No. 20 The use of BATS for INDRA.

*Delivery and commissioning in fact took place in February/March 1968.

Computer applications

OPERATIONAL RESEARCH

K. Wolfenden
‡ V. Calogero
‡ Dorothy Hickman
G. Mitra
† J. J. H. Forrest
† D. B. C. Richards
† H. C. Johnston

‡ grant-supported staff
† research student

Although for convenience labelled Operational Research, the interests of members of this group range widely from some of the computer techniques of operational research, such as mathematical programming, to the application of computers to civil engineering planning and design, and even to earthy data processing. Over the period of this report the main efforts have been in integer and dynamic programming, scheduling, and motorway design, as detailed below.

Linear programming

J. J. H. Forrest, who was investigating generalised upper bounding techniques and compact forms of the working basis in the solution of structural linear programming problems, was awarded a NATO Science Studentship and spent much of this period at the Operations Research Centre, University of California, Berkeley. However he left the Institute soon after his return.

Integer programming

D. B. C. Richards completed his work on integer programming and submitted his thesis in November 1967. This included original extensions to both the cutting plane and branch and bound methods. Extensive information on the solution of integer programs has also been derived from the behaviour of the various techniques as applied to a very wide range of test problems.

Non-linear programming

As well as being associated with the integer programming studies, G. Mitra has extended his interest in quadratic programming to include integer quadratic programs, and particularly those in which the integer variables may only take the values 0 or 1. A successful branch and bound algorithm has been developed for the fixed charge problem.

Tower spotting

Given the survey data of an overhead electric power transmission line route and a choice of the available suspension, tension and angle towers, a dynamic programming algorithm has been developed by Gautam Mitra and Keith Wolfenden for choosing and siting the towers (the location and angle of the angle towers being prescribed) in such a way that the overall cost of running the line from one end of the route to the other, subject to all the established design constraints, is a minimum. The program has been successfully run on design studies for the Central Electricity Generating Board.

Computer-aided construction of school and university teaching timetables

Despite their apparent similarity, from the computer standpoint the problems of constructing school and university timetables are quite distinct. H. C. Johnston has continued his work on an algorithm for the construction of a school timetable. A system for the presentation of all the relevant data has been developed. The school is described by a 2-dimensional model in which classes, teachers, rooms and other equipment are all regarded as 'items' which may have one or more 'lives'. A preliminary program checks the data and facilitates error detection. The main program seeks to find a timetable by choosing a series of assignments in such a way that the chance of failure is significantly reduced. If failure does occur, assignments are removed and alternative choices made. Trials are being conducted with a London comprehensive school.

Dorothy Hickman has written programs for the construction of university teaching timetables and these have been used experimentally within the University, which itself has provided special financial support for this work. Miss Hickman has also designed and implemented a special purpose timetabling language written for Atlas using the Compiler Compiler. This language has been designed to allow a quite general specification of the requirements

and constraints desired in a timetable. The number and kinds of items to be allocated (e.g. period, course, room) can be varied. All items or a specified number of disjoint or consecutive items from a set of items can be allocated or their allocation prohibited. Items can be requested or forbidden to precede or follow specified members of a set, while exceptions to a constraint can also be made. Items can be grouped into two kinds of sets; the first is a straightforward group of items (e.g. all geography classes, all periods on a Monday), while the second is a collection of such groups and is used when divisions exist between the members of a set, as for instance between the morning and afternoon hours of any day or between the different days of a week.

Computer-aided highway design

V. Calogero has completed a polynomial alignment program for the design and plotting of horizontal and vertical road alignments and the Ministry of Transport has issued recommendations and standards for the use of this new technique in the UK. V. Calogero has also developed a map processing system and has embarked on the addition of optimisation stages to his automatic design procedures. This project is supported in part by a grant from the Ministry of Transport.

PUBLICATIONS

K. Wolfenden and A. Wren.
Locomotive scheduling by computer. Proc. of 1966 *British Joint Computer Conference: IEE. Conf. Publ. No.* 19, *pp* 31–37.

G. Mitra and K. Wolfenden.
A computer technique for optimizing the sites and heights of transmission line towers–a dynamic programming approach. Comp. Journ. (1968) **10,** *pp* 347–51.

PH.D THESIS

D. B. C. Richards.
Integer programming–Theory and practice. University of London, 1968.

REPORTS

V. Calogero.
Polynomial alignment program. May 1966.

K. Wolfenden.
Sorting magnetic tape files. ICSI 107. *May* 1966.

G. Mitra/D. B. C. Richards and K. Wolfenden.
An improved algorithm for the solution of integer programs by the solution of associated diophantine equations. ICSP 105. *May* 1968. (*Also submitted to AFIRO for publication.*)

G. Mitra.
Dichotomizing procedure for the transportation problem. ICSI 105. *March* 1968.

G. Mitra.
Dichotomizing procedure for the fixed charge problem. ICSI 122. *June* 1968.

STATISTICAL RESEARCH

D. E. Barton
M. R. B. Clarke
† O. Abe
† A. Baruya
Anne Russel
Frances Cherry

† research student

This group, also in the initial stage of growth is concerned with the development of statistical theory and methodology, especially in the fields of multivariate analysis, time series, and combinatorial probabilities. Leading from this is an interest in the development of statistical computer systems of general applicability.

Statistical systems

From October 1967 onwards a preliminary study has been in progress into the needs of statistical systems, and the extent to which existing routines and data processing languages are suitable for integration into such a system. This entails fuller documentation of existing statistical advisory service programs, and their modification in the interests of compatibility and flexibility, with the aim of establishing a prototype system.

INTERNAL REPORTS

M. R. B. Clarke.
Principal components analysis. ICS/PS 18, *May* 1966.
M. R. B. Clarke and A. E. Maxwell.
Discriminant function (canonical variate analysis). ICS/PS 17, *May* 1966.
Split plot analysis of variance. ICS/PS 20, *May* 1966.

M.PHIL THESIS

O. Abe.
The random intersection of two multi-coloured graphs and statistical applications. University of London, 1967.

PUBLICATIONS

D. E. Barton and F. N. David.
Construction of graph-symmetric function tables. Proc. Int. Congr. Mathm. (Moscow). Abstr. Sci. Comm. (1966) **11,** *p* 3.
D. E. Barton and F. N. David.
A persistence problem in renewal theory. Biometrika (1966) **53,** *pp* 255–58.
D. E. Barton/F. N. David and M. G. Kendall.
Symmetric function and allied tables. Cambridge University Press, 1966.
D. E. Barton.
Comparison of sequential binomial proportions. Technometrics (1967) **9,** *pp* 337–79.
D. E. Barton *et al.*
Tests for space-time interaction and a power function. Proc. Fifth Berkeley Symp. (1967) **5,** *pp* 217–27.
D. E. Barton *et al.*
A review of the analysis of karyographs of the human cell in mitosis. Proc. Fifth Berkeley Symp. (1967) **5,** *pp* 349–66.

MISCELLANEOUS APPLICATIONS

During the period up to the end of 1967 a variety of computer applications have been studied at the Institute which do not readily fall under the main headings of this report. These are described below, with the names of the staff involved in each case.

A library of crystallographic programs

Jean Dollimore

A comprehensive system of programs for the solution of crystallographic problems is now available as a result of about two years effort. The aim was to produce a system in which a user could introduce data and call programs according to simple conventions. This has been developed using a method which is adaptable to any block structure compiler. Each crystal library program consists of definitions of named locations for single items of data, and of routines for input, calculation and output. The user's program forms the end of this program and consists of instructions to store values in these locations, to call for the standard routines and also for the user's own routines. The multiple data is input from one of four types of list designed for crystallographic data. All calculation routines search for data and store results in the computer according to the arguments specified with their calls. These programs are loaded in compiled form on a magnetic tape with master routines which read the user's programs (excluding user's own routines). When it became possible to load open-ended programs on Atlas an alternative version was developed (by Roy Baker) in which users could specify their own routines, and this is also loaded on the tape. The system could be improved still further by the use of magnetic tape as an intermediate store between runs.

PUBLICATION

J. Dollimore.
A general Fourier synthesis program for the London University Atlas computer. Comput. J. (1966) **8,** *pp* 347–51.

TECHNICAL NOTES

J. Dollimore.
*A Fourier program for LUNA. LSP*6 *February* 1966 *and LSP*32 *November* 1966.
A note on allocation of core space. LSP 8, *February* 1966.
A least squares program for LUNA. LSP 12, *March* 1966 *and LSP* 62, *January* 1968.
Some crystallographic data processing routines. LSP 17, *April* 1966.
Data lists for crystal library programs. LSP 45, *May* 1967.
A structure factors program for LUNA. LSP 47, *June* 1967.
A distance angle program for LUNA. LSP 48, *June* 1967.
Some crystallographic data processing routines. LSP 68, *February* 1968.

Computer animation

W. L. B. Nixon
A. G. M. Pritchett
Margaret J. Pragnell

This project has studied the feasibility of using computers for the production of animated pictures similar to cartoon films, which are normally made using single-shot shutter control from drawings on transparent overlays. The ultimate aim is to put a computer system with graphical-output display and recording facilities at the disposal of the creative artist; but it became clear that with the computer techniques currently available, the tedious and time-consuming film method has an overwhelming advantage in respect of cost. Some advance has nevertheless been made towards the design of suitable data-structures for computer storage and manipulation of animation information; and a very short cartoon film generated on ATLAS and transferred off-line to film has been made.

Print layout and design

R. J. Wakefield

A preliminary investigation has been made into the feasibility of using a computer equipped with a CRT display for layout and design work. This was mainly concerned to find out whether a CRT system could create images of sufficient typographic standard, whilst allowing the size of face to be such that a realistic layout of a page could be represented at any one time.

PUBLICATIONS

R. J. Wakefield.
Print layout and design with a computer CRT system. J. Typogr. Res. (1967) **1**, *pp* 165–68.

Applications of the perceptual maze test

A. Elithorn (Royal Free Hospital)
† D. N. Lee
† J. R. Jagoe
† research student

The perceptual maze test is a psychological intelligence test based on a triangular lattice containing a random distribution of dots at the intersections. The task of the subject is to find a path through this lattice, starting at the vertex, which passes through the maximum number of dots. Performance in the test is relatively free from educational bias, and it is considered that the basic components of the solving processes used are perceptual integrating skills which are fundamental to intelligent behaviour. These are thought to be more easily impaired by organic brain damage than skills mainly acquired by training, hence the test may be a valuable tool for comparing brain-damaged and normal subjects.

D. N. Lee has shown that the structure of the task is amenable to formal treatment using a graph-theoretical matrix model. The structural properties of a particular maze pattern can be expressed

as numerical parameters, in terms of which the difficulty of solution can be studied and so lead to an understanding of the solving processes used by subjects. An extended experiment has been carried out using 72 maze patterns, which were generated by computer, and 288 subjects (144 men and 144 women, all RAF recruits).

A detailed analysis of the experimental results has been given in the PH.D thesis by Lee and in later papers. These include deductions about the type of solving process used, a systematic study of differences of performance attributable to sex and two different methods of presenting the test problems, and the effects of practice on performance.

The method appears to be a fruitful method of studying human problem solving. Lee has also done preliminary work on a model simulating the solving process on a computer. His model has been extended and explored in greater detail by J. R. Jagoe.

PH.D THESIS

D. N. Lee.
A psychological and mathematical study of task complexity in relation to human problem-solving using a perceptual maze test. University of London, 1965.

PUBLICATIONS

R. A. Buckingham/A. Elithorn/D. N. Lee and W. L. B. Nixon.
A mathematical model of a perceptual maze test. Nature (1963) **199,** *pp* 676–78.
A. Elithorn/D. Jones/M. Kerr and D. N. Lee.
The effects of the variation of two physical parameters on empirical difficulty in a perceptual maze test. Brit. J. Psychol (1964) **55,** *pp* 31–37.
A. Elithorn/J. R. Jagoe and D. N. Lee.
The simulation of a perceptual problem-solving skill. Nature (1966) **211,** *pp* 1029–31.
D. N. Lee/G. Jessup and A. Elithorn.
Pattern parameters determining item difficulty in a perceptual maze test. (1966.)

Transport properties of dilute gases

R. A. Buckingham
J. W. Fox (University College, London)
E. Gal

Extensive calculations have been carried out on low energy collisions between hydrogen atoms, using quantal scattering theory and the accurate interaction potentials derived by Dalgarno and Lynn. The wave equation has been integrated numerically, and asymptotic phase-shifts derived for values of the azimuthal quantum number from 0 to 35. This has made it possible to derive accurately the quantal cross-sections for viscosity and thermal conductivity of atomic hydrogen from 1° to 400°K; to compare these results with approximate methods of calculation and with experimental data. The very interesting behaviour of the phase-shifts associated with the ground state singlet interaction of hydrogen atoms has

been analysed in detail. Comprehensive computer programs are now available for use with other forms of intermolecular potential.

This work has also enabled elastic scattering cross-sections to be derived for collisions of hydrogen atoms, with various assumptions about the symmetization of the wave functions, or proton identity effects. These cross-sections have been computed for collision energies up to 0.37 electron-volt.

PUBLICATIONS

R. A. Buckingham/J. W. Fox and E. Gal.
The coefficients of viscosity and thermal conductivity of atomic hydrogen from 1 *to* 400°*K. Proc. Roy. Soc.* (1965) **A 284,** *pp* 237–51.
J. W. Fox and E. Gal.
Elastic cross-sections of hydrogen atoms. Proc. Phys. Soc. (1967) **90,** *pp* 55–61.

Eddy-current losses

M. J. M. Bernal
J. M. Bullingham (University College London)

Numerical methods of an original kind have been used to calculate eddy-current losses in ferromagnetic laminations, taking account of hysteresis and non-linearity in the B/H loops. The results confirm that these effects contribute only a small part of the anomalous losses which are observed experimentally.

PUBLICATION

J. M. Bullingham and M. J. M. Bernal.
Investigation of the effect of non-linear B/H loops on the calculation of eddy current losses. Proc. IEE (1967) **114,** *pp* 1174–76.

Propagation of plane waves in solids

G. J. Cooper
J. W. Craggs (University of Melbourne)

The equations representing the propagation of plane waves in isotropic plastic solids, based on a suitable model, have been solved numerically for a semi-infinite plate using a finite difference approximation, and the same model has also been applied to wave propagation in an elastic plate. With some simple refinements in the standard numerical techniques good agreement has been obtained between the mathematical model and experimental results for plastic solids.

PUBLICATIONS

G. J. Cooper and J. W. Craggs.
Propagation of plane waves in plastic solids. J. Aust. Math. Soc. (1965) *V, pp* 349–64.
Propagation of elastic waves. J. Aust. Math. Soc. (1966) *VI, pp* 55–64.
A note on the propagation of plane waves in elastic and plastic solids. J. Aust. Math. Soc. (1968) *VIII, pp* 231–37.

Numerical analysis and methodology

M. J. M. Bernal
W. G. Bickley
G. J. Cooper
E. Gal
C. C. Paige
K. E. Pitman
† A. Bradwell
† D. J. Fyfe
† W. S. Murray
* G. L. Parker
† G. Pugh
† R. Thatcher
J. H. Verner
† J. R. Whiteman
† P. W. Williams

* short-term assistant
† research students

Broadly speaking the interest of the group is numerical analysis in all its aspects. The research already completed or in progress is mainly concerned with the numerical solution of ordinary or partial differential equations and of integral equations, and with linear algebra.

Theory of approximation

(a) Approximation by Chebyshev polynomials

An arbitrary continuous function may be approximated over a finite range by a truncated Chebyshev series, the coefficients being obtained by numerical quadrature using the trapezoidal rule. G. J. Cooper has shown how the better known methods for forming these coefficients may be refined by using some unfamiliar methods of harmonic analysis to give improved accuracy and to reduce the amount of computation.

(b) Errors of polynomial approximation

G. J. Cooper has also obtained bounds for errors in polynomial interpolation to a function by using properties of the function in the complex plane.

Non-linear equations

G. Pugh has begun a study of methods for the solution of non-linear equations.

Constrained optimization

It is possible to transform the problem of constrained optimization into that of a sequence of unconstrained problems. Such a sequence of problems presents certain difficulties for conventional algorithms. W. S. Murray has been investigating the precise nature of these difficulties and how they may be overcome.

Solution of ordinary differential equations

(a) Single-step methods

G. J. Cooper and E. Gal have examined a single-step process of Runge-Kutta type for a linear differential equation of a given order. They have derived conditions which constrain the parameters of the process and which are necessary to give methods of specified order. A simple set of sufficient conditions has also been obtained. G. J. Cooper has obtained general implicit single-step methods for the numerical solution of a system of non-linear differential equations of arbitrary order and has investigated their convergence properties.

J. H. Verner has determined sufficient conditions for a method of this type to be of a given order, and has obtained expressions for the leading error terms.

(b) Interpolation and quadrature methods

G. J. Cooper has proposed a class of single step methods for differential equations based on the use of interpolation and quadrature formulae and related to implicit methods of Runge-Kutta type.

(c) Solution of equations with a singularity

Conditions necessary for the convergence of difference schemes for a large class of second order ordinary differential equations with singularities at one end point of the domain of definition have been investigated by J. H. Verner, and sufficient conditions for convergence in a sub-class obtained; it is not known whether these conditions can be generalised to include the whole class of problems considered. In the report on this investigation numerical results for a large number of examples are given.

(d) Two-point boundary problems

Prof. W. G. Bickley has been investigating the use of cubic splines in the solution of two-point boundary value problems for ordinary differential equations. The comparison with analytical solutions is encouraging. Comparison has also been made with the application of Hermite interpolation, which gives results of the same order of accuracy but generally involves the solution of many more linear equations for the same number of nodes. This work is being extended by D. J. Fyfe.

Eigenvalues of large sparse matrices

Several iterative methods are available for finding a few roots and vectors of a large sparse positive definite matrix. As yet there has been little interest in the same problem for unsymmetric matrices. C. C. Paige is examining various gradient methods in order to evaluate their applicability and efficiency.

Partial differential equations

(a) Theory of successive over-relaxation

A number of distinct, but closely related, approaches to SOR theory have been developed in the past few years. In the present work by M. J. M. Bernal and J. H. Verner a new definition of consistent ordering has been proposed which includes the scope of recent definitions, and corresponding generalizations of SOR theory have been obtained. The facility with which the new definition can be applied has been demonstrated.

(b) Problems in plane elasticity

K. E. Pitman has been examining in detail the application of the method of Davis and Rabinowitz to the solution of biharmonic boundary value problems arising in plane elasticity. Use has been made of the complex potentials introduced by Muschelvili. Satisfactory results have been obtained for a variety of problems including instances when singularities have to be removed analytically and for mixed boundary value problems; in some of the latter cases the transition points were initially unknown and have been closely located. This work is being written up as a PH.D thesis and it is hoped to publish some of the more interesting results.

(c) Boundary value problems

A flexible program for the application of point and block successive over-relaxation and Chebyshev iterative methods to elliptic boundary value problems has been developed by C. C. Paige.

Methods for the numerical treatment of singularities occurring in elliptic boundary value problems have been investigated by M. J. M. Bernal and J. R. Whiteman. They have generalized the methods of Motz and Woods and applied these generalizations to biharmonic boundary value problems. Whiteman has developed a dual series technique for the solution of problems of this type and has obtained error bounds. Mr Whiteman has completed his PH.D thesis on this work.

R. Thatcher has begun an investigation of the application of variational methods to the solution of problems in elastomechanics. The scope of the investigation includes the determination of error bounds, convergence problems, the treatment of singularities and developments of the finite element technique.

(d) Parabolic equations

D. J. Fyfe has been studying the application of generating function techniques to the numerical solution of parabolic partial differential equations. Interesting results have been obtained and these will

probably appear as an Institute report with a view to ultimate publication.

(e) Stability in solution of differential equations

P. W. Williams has been developing a unified approach to stability for both ordinary and partial differential equations which is not restricted to the linear case. The work puts the theories of Dahlquist, Godounov and Ryabenki and of Lax and Richtmyer in a common framework and is fairly closely related to the work of Stetter. This approach is being used in an attempt to rectify limitations of some of the existing theories.

INTERNAL REPORTS

G. J. Cooper.
Errors in polynomial approximation. May 1967.
G. J. Cooper and E. Gal.
Implicit Runge-Kutta methods for the solution of linear differential equations. ICSP 100. *August* 1967 (*submitted for publication*).
J. H. Verner.
Convergence of a finite difference scheme for a second order ordinary differential equation with a singularity. ICS 001. 1967.
J. H. Verner.
The order of some implicit Runga-Kutta methods. ICSP 101. *September* 1967.
M. J. M. Bernal and J. R. Whiteman.
Singularities due to re-entrant corners in harmonic boundary-value problems. Issued as Technical Summary Report 829 *of Mathematics Research Centre, Madison, Wisconsin. December* 1967.

PH.D THESIS

J. R. Whiteman.
Singularities in the solution of harmonic and biharmonic boundary problems. University of London, 1967.

PUBLICATIONS

G. J. Cooper.
The evaluation of coefficients in a Chebyshev expansion. Computer J. (1967) **10,** *pp* 94–100.
G. J. Cooper and E. Gal.
Single-step methods for linear differential equations. Num. Math. (1967) **10,** *pp* 307–15.
G. J. Cooper.
A class of single-step methods for systems of non-linear differential equations. Math. Comp. (1967) **21,** *pp* 597–610.
G. J. Cooper.
Interpolation and quadrature methods for ordinary differential equations. Math. Comp. (1967) **22,** *pp* 69–76.
J. R. Whiteman.
Treatment of singularities in harmonic mixed boundary value problem by dual series methods. Q.J. Mech. Appl. Math. (1968) **21,** *pp* 41–50.
J. H. Verner and M. J. M. Bernal.
On generalizations of the theory of consistent orderings for successive over-relaxation methods. Num. Math. (1968) **12,** *pp* 215–22.
W. G. Bickley.
Piecewise cubic interpolation and two-point boundary problems. Comput. J. (1968) **11,** *pp* 206–08.

Research degrees awarded to Institute students up to December 1967

Ph.D

1965	J. M. Gerard	Synthesizing non-linear forms from linear descriptions by syntactic analysis.
1965	A. W. Sambles	On the mechanization of algebraic manipulation by digital computer.
1965	D. N. Lee	A psychological and mathematical study of task complexity in relation to human problem-solving using a perceptual maze text.
1966	I. B. Parker	Studies in the approximate solution of partial differential equations of parabolic type.
1967	J. R. Whiteman	Singularities in the solution of harmonic and biharmonic boundary problems.

M.Phil

1967	O. Abe	The random intersection of two multi-coloured graphs and statistical applications.